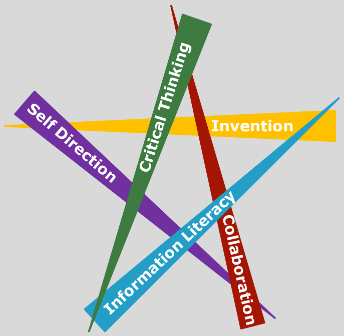
**Curriculum Development Course at a Glance**
**Planning for High School Mathematics**

| Content Area | Mathematics | Grade Level | High School |
|---|---|---|---|
| **Course Name/Course Code** | Integrated Math 3 | | |

| Standard | Grade Level Expectations (GLE) | GLE Code |
|---|---|---|
| 1. Number Sense, Properties, and Operations | 1. The complex number system includes real numbers and imaginary numbers | MA10-GR.HS-S.1-GLE.1 |
| | 2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations | MA10-GR.HS-S.1-GLE.2 |
| 2. Patterns, Functions, and Algebraic Structures | 1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables | MA10-GR.HS-S.2-GLE.1 |
| | 2. Quantitative relationships in the real world can be modeled and solved using functions | MA10-GR.HS-S.2-GLE.2 |
| | 3. Expressions can be represented in multiple, equivalent forms | MA10-GR.HS-S.2-GLE.3 |
| | 4. Solutions to equations, inequalities and systems of equations are found using a variety of tools | MA10-GR.HS-S.2-GLE.4 |
| 3. Data Analysis, Statistics, and Probability | 1. Visual displays and summary statistics condense the information in data sets into usable knowledge | MA10-GR.HS-S.3-GLE.1 |
| | 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions | MA10-GR.HS-S.3-GLE.2 |
| | 3. Probability models outcomes for situations in which there is inherent randomness | MA10-GR.HS-S.3-GLE.3 |
| 4. Shape, Dimension, and Geometric Relationships | 1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically | MA10-GR.HS-S.4-GLE.1 |
| | 2. Concepts of similarity are foundational to geometry and its applications | MA10-GR.HS-S.4-GLE.2 |
| | 3. Objects in the plane can be described and analyzed algebraically | MA10-GR.HS-S.4-GLE.3 |
| | 4. Attributes of two- and three-dimensional objects are measurable and can be quantified | MA10-GR.HS-S.4-GLE.4 |
| | 5. Objects in the real world can be modeled using geometric concepts | MA10-GR.HS-S.4-GLE.5 |

## Colorado 21st Century Skills




**Critical Thinking and Reasoning:** *Thinking Deeply, Thinking Differently*

**Information Literacy:** *Untangling the Web*

**Collaboration:** *Working Together, Learning Together*

**Self-Direction:** *Own Your Learning*

**Invention:** *Creating Solutions*

## Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| Unit Titles | Length of Unit/Contact Hours | Unit Number/Sequence |
|---|---|---|
| Survey Says… | 3 weeks | 1 |
| Within and Around | 6 weeks | 2 |
| Let Poly be Rational | 6 weeks | 3 |
| Sine of the Times | 5 weeks | 4 |
| Function App | 6 weeks | 5 |

**Curriculum Development Overview**
**Unit Planning for High School Mathematics**

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|---|---|---|---|
| **Unit Title** | Survey Says | **Length of Unit** | 3 weeks |
| **Focusing Lens(es)** | Justification<br>Inferences | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.HS-S.3-GLE.1<br>MA10-GR.HS-S.3-GLE.2 |
| **Inquiry Questions (Engaging-Debatable):** | • When should sampling be used? When is sampling better than a census? (MA10-GR.HS-S.3-GLE.2-IQ.3) | | |
| **Unit Strands** | Statistics and Probability: Making Inferences and Justifying Conclusions<br>Statistics and Probability: Interpreting Categorical and Quantitative Data | | |
| **Concepts** | inferences, parameters, random sample, population, validity, sampling, surveys, experiments, observational studies, statistical results, randomization, simulation, indirect, data | | |

| **Generalizations**<br>**My students will Understand that…** | **Guiding Questions**<br>**Factual** | **Conceptual** |
|---|---|---|
| Random samples from a population allow statisticians to make inferences about population parameters. (MA10-GR.HS-S.3-GLE.2.a) | How can we reduce the margin of error in a population prediction?<br>How can I use mean and standard deviation of a data set to draw a normal distribution?<br>What happens to sample-to-sample variability when you increase the sample size? (MA10-GR.HS-S.3-GLE.2-IQ.2) | Why is the normal distribution commonly used to model a population and when is this not appropriate?<br>How can the results of a statistical investigation be used to support an argument? (MA10-GR.HS-S.3-GLE.2-IQ1)<br>Why is the margin of error in a study important? (MA10-GR.HS-S.3-GLE.2-IQ.5)<br>How is it known that the results of a study not simply due to chance? (MA10-GR.HS-S.3-GLE.2-IQ.6) |
| Validity in sampling, surveys, experiments, observational studies and the interpretation of statistical results depends on randomization. (MA10-GR.HS-S.3-GLE.2-EO.b) | In what ways can a survey be biased?<br>How does randomization factor into the design of an experiment? | Why is randomization an important component of sampling? |
| Simulation provides a means to indirectly collect data. (MA10-GR.HS-S.3-GLE.2-EO.b) | How do you design a simulation to model the collection of data that isn't easily obtainable? | How has the use of technology enhanced our ability to study difficult to measure phenomena? |

| **Key Knowledge and Skills:** **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* |
|---|---|

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (MA10-GR.HS-S.3-GLE.2-EO.a.i)
- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (MA10-GR.HS-S.3-GLE.2-EO.a.ii)
- Recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each. (MA10-GR.HS-S.3-GLE.2-EO.b.i)
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (MA10-GR.HS-S.3-GLE.2-EO.b.ii, iii)
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (MA10-GR.HS-S.3-GLE.2-EO.b.iv)
- Evaluate reports based on data. (MA10-GR.HS-S.3-GLE.2-EO.b.vi)
- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and recognize that there are data sets for which such a procedure is not appropriate; use calculators, spreadsheets, and tables to estimate areas under the normal curve. (MA10-GR.HS-S.3-GLE.1-EO.a.iv, v)

**Critical Language:** includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.
EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *“Mark Twain exposes the hypocrisy of slavery through the use of satire.”*

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| **A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):** | *Statistics is a process for making inferences about population parameters based on a random sample of a population.* |
| **Academic Vocabulary:** | inferences, surveys, experiments, observational studies, data, conclusions, interpret, evaluate, recognize, compare, model, explain, spreadsheets, estimate |
| **Technical Vocabulary:** | statistic, statistics, sampling, mean, standard deviation, data sets, normal distribution, normal curve, margin of error, parameters, random sample, population, validity, sampling, statistical results, randomization, simulation, indirect, data |

| **Unit Title** | Within and Around | | **Length of Unit** | 6 weeks |
|---|---|---|---|---|
| **Focusing Lens(es)** | Perspective<br>Interdependence | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.HS-S.4-GLE.1<br>MA10-GR.HS-S.4-GLE.2<br>MA10-GR.HS-S.4-GLE.3 | |
| **Inquiry Questions (Engaging-Debatable):** | • Do perfect circles naturally occur in the physical world? If so, how do we model them? (MA10-GR.HS-S.4-GLE.2-IQ.4)<br>• Why are circles at the foundation of geometric constructions? | | | |
| **Unit Strands** | Geometry: Circles<br>Geometry: Expressing Geometric Properties with Equations<br>Geometry: Congruence | | | |
| **Concepts** | arc length, inscribed angles, circumscribed angles, central angles, circles, center, radius, equation, chords, arcs, proportionally, proofs, geometric constructions, conjecture, coordinate plane, geometric relationships | | | |

| **Generalizations**<br>**My students will Understand that…** | **Guiding Questions**<br>**Factual** | **Conceptual** |
|---|---|---|
| Arc length determines the interdependent relationship of inscribed, circumscribed and central angles of a circle. (MA10-GR.HS-S.4-GLE.2-EO.e) | What is the relationship between inscribed, central, and circumscribed angles of a circle that subtend to the same arc?<br>How does the measure of the central angle help you find the area of the corresponding sector? | Why are inscribed, central, and circumscribed angles of a circle independent with each other when they subtend the same arc? |
| The center and radius of the circle constrain the equation by providing location and size. (MA10-GR.HS-S.4-GLE.3-EO.a.i.1, 2) | What is equation of a circle?<br>Within the equation of the circle, where is the center and the radius? | How does the Pythagorean Theorem define all points on a circle with a given center and radius?<br>Why is the radius of a circle perpendicular to the tangent where the radius intersects the circle? |
| The length of chords and their corresponding arcs vary proportionally. (MA10-GR.HS-S.4-GLE.2-EO.f) | What is the longest chord in a circle and how do you know? | Why does a radius that bisects an arc also bisect the corresponding chord? |

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| Geometric constructions create a visual proof by showing a logical progression of statements that prove or disprove a conjecture. (MA10-GR.HS-S.4-GLE.1-EO.a.vi, d.i) | What is formal geometric construction?<br>How does a geometric construction differ from a geometric drawing or sketch?<br>How does the construction of a perpendicular bisector of a line segment help prove that all the points on the bisector are equidistant from the endpoints of the segment?<br>How does the construction of the medians of a triangle help prove they will always meet at a point? | How does a geometric construction connect to terms and definitions? |
| The coordinate plane models algebraically two-dimensional geometric relationships. (MA10-GR.HS-S.4-GLE.3-EO.a.ii) | What information is needed to calculate the perimeters of polygons and area of triangles and rectangles in the coordinate plane?<br>How can you determine the slope of line parallel or perpendicular to a given line? | Why is it helpful to model geometric relationships on the coordinate plane?<br>How can the relationship between area and volume be explained through cross-sections and rotations? |

| **Key Knowledge and Skills:** **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* |
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- Prove that all circles are similar (MA10-GR.HS-S.4-GLE.2-EO.b.i)
- Identify and describe relationships among inscribed angles, radii, and chords. (MA10-GR.HS-S.4-GLE.2-EO.e.i)
- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (MA10-GR.HS-S.4-GLE.2-EO.e.ii, iii)
- Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (MA10-GR.HS-S.4-GLE.2-EO.f)
- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (MA10-GR.HS-S.4-GLE.3-EO.a.i.1, 2)
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (MA10-GR.HS-S.4-GLE.1-EO.d.ii)
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (MA10-GR.HS-S.4-GLE.4-EO.b.i)
- Make formal geometric constructions with a variety of tools and methods. (MA10-GR.HS-S.4-GLE.1-EO.d.i)
- Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (MA10-GR.HS-S.4-GLE.3-EO.a.ii.3)
- Use the distance formula on coordinates to compute perimeters of polygons and areas of triangles and rectangles. (MA10-GR.HS-S.4-GLE.3-EO.a.ii.4)
- Use coordinates to prove simple geometric theorems algebraically. (MA10-GR.HS-S.4-GLE.3-EO.a.ii.1)
- Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. (MA10-GR.HS-S.4-GLE.3-EO.a.ii.2)

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| **A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):** | *The central angle of a circle is twice the measure of its corresponding inscribed angle.* |

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| **Academic Vocabulary:** | prove, construct, derive, area, equilateral triangle, square, regular hexagon, distance, angle, conjecture, point, circle, define, represent, compare, develop |
| **Technical Vocabulary:** | arc length, inscribed angles, circumscribed angles, central angles, circles, center, radius, equation, chords, arcs, proportionally, sector, diameter, perpendicular, tangent, quadrilateral, equation, bisect, similarity, proofs, geometric constructions, conjecture, coordinate plane, geometric relationships |

| **Unit Title** | Let Poly be Rational | | **Length of Unit** | 6 weeks |
|---|---|---|---|---|
| **Focusing Lens(es)** | Transformations<br>Structure | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.HS-S.2-GLE.1<br>MA10-GR.HS-S.2-GLE.3<br>MA10-GR.HS-S.2-GLE.4 | |
| **Inquiry Questions (Engaging- Debatable):** | • What is the square root of negative 1? What are the implications of having a solution to this problem?<br>• How did the ancient Greeks multiply binomials and find roots of quadratic equations without algebraic notations? (MA10-GR.HS-S.2-GLE.3-IQ.2) | | | |
| **Unit Strands** | Algebra: Creating Equations<br>Algebra: Arithmetic with Polynomials and Rational Expressions<br>Algebra: Seeing Structure in Expressions<br>Algebra: Reasoning with Equations and Inequalities<br>Functions: Interpreting Functions<br>Geometry: Expressing Geometric Properties with Equations | | | |
| **Concepts** | polynomial expression, polynomial equations, polynomials, closed system, properties of operations, rational expressions, rational equations, radical equations, focus, directrix, parabola, equations, expressions, solutions, extraneous solutions, zeros | | | |

| **Generalizations**<br>**My students will Understand that…** | **Guiding Questions**<br>**Factual** | **Conceptual** |
|---|---|---|
| The transformation of polynomial expressions and equations can reveal underlying structures and solutions. (MA10-GR.HS-S.2-GLE.3-EO.a, d, e) | How is factoring used to solve a polynomial with a degree greater than two?<br>When is it appropriate to simplify expressions? (MA10-GR.HS-S.2-GLE.3-IQ.1) | How can polynomial identities be used to describe numerical relationships?<br>Why is the remainder theorem useful?<br>How are factors of polynomials connected to zeros of polynomials and solutions of polynomial equations? |
| Polynomials form a closed system under the operations of addition, subtraction, and multiplication analogous to the integers. (MA10-GR.HS-S.2-GLE.3-EO.c.i) | What operations can be done to two polynomials that will result in another polynomial?<br>How are rational and irrational numbers similar and different from integers with respect to closure? | Why is it important to know that polynomials are closed under these operations? |
| Properties of operations transform rational expressions with the intention of creating more efficient forms of the expression. (MA10-GR.HS-S.2-GLE.3-EO.g) | How can inspection, long division and computer algebra systems be used to rewrite rational expressions?<br>How do you use factoring to rewrite a rational expression? | Why do we rewrite rational expressions in different forms?<br>Why can computers solve problems that humans cannot? (MA10-GR.HS-S.2-GLE.4-IQ.3) |
| Solving rational and radical equations can result in extraneous solutions. (MA10-GR.HS-S.2-GLE.4-EO.b.ii) | How do you check for extraneous solutions?<br>When do extraneous solutions arise?<br>How can you determine if a solution is not viable? | Why do extraneous solutions occur? |

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| Mathematicians use the focus and directrix of a parabola to derive an equation. (MA10-GR.HS-S.4-GLE.3-EO.a.3) | How can you derive a quadratic equation from a focus and directrix? | Why does the focus and directix define a parabola? |

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| **Key Knowledge and Skills:** **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* |

- Understand polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication (MA10-GR.HS-S.2-GLE.3-EO.c.i)
- Add, subtract, and multiply polynomials. (MA10-GR.HS-S.2-GLE.3-EO.c.i)
- Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. (MA10-GR.HS-S.2-GLE.3-EO.d.i)
- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (MA10-GR.HS-S.2-GLE.1-EO.c.iv)
- State and apply the remainder theorem. (MA10-GR.HS-S.2-GLE.3-EO.d.i)
- Identify zeros of quadratic, cubic, and quartic polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (MA10-GR.HS-S.2-GLE.3-EO.d.ii)
- Prove polynomial identities and use them to describe numerical relationships. (MA10-GR.HS-S.2-GLE.3-EO.e.i)
- Use the structure of a polynomial, rational or exponential expression to identify ways to rewrite it. (MA10-GR.HS-S.2-GLE.3-EO.a.ii)
- Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. (MA10-GR.HS-S.2-GLE.3-EO.g)
- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (MA10-GR.HS-S.2-GLE.4-EO.b.ii)
- Explain each step in solving simple rational or radical equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution and construct a viable argument to justify a solution method. (MA10-GR.HS-S.2-GLE.4-EO.b.i)
- Create simple rational equations and exponential and inequalities in one variable and use them to solve problems. (MA10-GR.HS-S.2-GLE.4-EO.a.i)
- Derive the equation of a parabola given a focus and directrix. (MA10-GR.HS-S.4-GLE.3-EO.a.3)

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| **A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):** | *By factoring a polynomial function the zeros can be calculated and used, along with the end behavior, to construct a rough graph of the function.* |
| **Academic Vocabulary:** | solve, graph, identify, prove, rewrite, equality, solve, explain, create, state, apply, structure, construct, argument, justify, method |
| **Technical Vocabulary:** | polynomial expression, polynomial equations, polynomials, closed system, properties of operations, rational expressions, rational equations, radical equations, focus, directrix, parabola, equations, expressions, solutions, extraneous solutions, zeros, end behavior, factor, factorization, degree, derive, polynomial identities, functions, Remainder theorem, radicals, inspection, long division, quotient, remainder, divisor, degree of polynomial |

| **Unit Title** | Sine of the Times | | **Length of Unit** | 5 weeks |
|---|---|---|---|---|
| **Focusing Lens(es)** | Relationships<br>Modeling | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.HS-S.2-GLE.1<br>MA10-GR.HS-S.2-GLE.2<br>MA10-GR.HS-S.4-GLE.2 | |
| **Inquiry Questions (Engaging-Debatable):** | • How does the periodicity in the unit circle correspond to the periodicity in graphs of models of periodic phenomena? (MA10-GR.HS-S.2-GLE.2-EO.c)<br>• Why can the same class of functions model diverse types of situations (e.g., sales, manufacturing, temperature, and amusement park rides)? | | | |
| **Unit Strands** | Functions: Interpreting Functions<br>Functions: Trigonometric Functions | | | |
| **Concepts** | unit circle, coordinate plane, trigonometric functions, angles, model, periodic phenomena | | | |

| **Generalizations**<br>**My students will Understand that…** | **Guiding Questions**<br>**Factual** | **Conceptual** |
|---|---|---|
| The unit circle in the coordinate plane represents the trigonometric functions for any angle. (MA10-GR.HS-S.2-GLE.1-EO.f.ii), (MA10-GR.HS-S.2-GLE.4-EO.d) and (MA10-GR.HS-S.4-GLE.2-EO.d) | How is the circumference of a unit circle used to determine the radian measure of an angle?<br>Given an angle, how is the unit circle used to determine each of the trigonometric functions?<br>How are the relationships of right triangles used to determine the trigonometric functions of an angle? | How is the Pythagorean identity represented in the unit circle?<br>How does the Pythagorean Identity illustrate the inverse nature of the relationship between sine and cosine? |
| Trigonometric functions model periodic phenomena. (MA10-GR.HS-S.2-GLE.2-EO.c.i, iv) | What situations would it be appropriate to model with trigonometric?<br>How are frequency, midline and amplitude reflected in the equation of a trigonometric function? | Why would the parent trigonometric function change in period, midline and amplitude for a given situation? |

| **Key Knowledge and Skills:** **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* |
|---|---|

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. ((MA10-GR.HS-S.2-GLE.1-EO.f.i)
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (MA10-GR.HS-S.2-GLE.1-EO.f.ii)
- Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. (MA10-GR.HS-S.2-GLE.2-EO.c.i)
- Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. (MA10-GR.HS-S.2-GLE.4-EO.d)

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|---|---|
| **A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):** | *I know the sine of an angle in the first two quadrants of the unit circle is always positive.* |
| **Academic Vocabulary:** | explain, prove, graph, key features, interpret, angles, model, counterclockwise, clockwise, |
| **Technical Vocabulary:** | unit circle, coordinate plane, trigonometric functions, periodic phenomena, radian measure, subtend, amplitude, frequency, midline, period, Pythagorean identity, sine, cosine, tangent, arc length, real numbers, quadrant |

| **Unit Title** | Function App | | **Length of Unit** | 6 weeks |
|---|---|---|---|---|
| **Focusing Lens(es)** | Structure<br>Visualization | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.HS-S.1-GLE.2<br>MA10-GR.HS-S.2-GLE.1<br>MA10-GR.HS-S.2-GLE.2<br>MA10-GR.HS-S.2-GLE.3<br>MA10-GR.HS-S.2-GLE.4 | |
| **Inquiry Questions (Engaging-Debatable):** | • Why are functions necessary to the design and building of skyscrapers? (MA10-GR.HS-S.2-GLE.1-IQ.7) | | | |
| **Unit Strands** | Number and Quantity: Quantities<br>Algebra: Creating Equations<br>Algebra: Reasoning with Equations and Inequalities<br>Algebra: Seeing Structure in Expressions<br>Functions: Interpreting Functions<br>Functions: Building Functions<br>Functions: Linear, Quadratic, and Exponential Models | | | |
| **Concepts** | functions, nonlinear, model, graph, visualization, key features, domain, range, maxima, minima, intercepts, symmetry, end behavior, average rate of change, rate, systems of nonlinear functions, solutions, linear functions, logarithms, exponential, inverse | | | |

| **Generalizations**<br>**My students will Understand that…** | **Guiding Questions**<br>**Factual** | **Conceptual** |
|---|---|---|
| The modeling of nonlinear relationships between two quantities requires the use of appropriate functions. (MA10-GR.HS-S.1-GLE.2-EO.a.ii), (MA10-GR.HS-S.2-GLE.3-EO.b.ii) and (MA10-GR.HS-S.2-GLE.4-EO.a.ii) | How can you determine from a table or context, which function models the relationship between two quantities?<br>What phenomena can be modeled with particular functions? (MA10-GR.HS-S.2-GLE.2-IQ.2)<br>What are residuals and how do I calculate them? | How do you use the key features of families of functions to determine the appropriate function for a given situation?<br>Why do we classify functions? (MA10-GR.HS-S.2-GLE.2-IQ.1)<br>Why is a geometric series modeled with an exponential function? |
| Mathematicians can predict the effects on a graph when replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). (MA10-GR.HS-S.2-GLE.1-EO.e.i) | What is the impact of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative)? | Why are the effects on a graph predictable when replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative)? |

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| The visualization of a variety of functions on a coordinate plane helps to interpret of key features, such as domain, range, maxima, minima, intercepts, symmetry, end behavior and average rate of change. (MA10-GR.HS-S.2-GLE.1-EO.b.i, c) | What are important characteristics of a function that can be seen on a graph?<br>What do the graphs of linear, exponential, square root, cube root, step and absolute value functions look like?<br>How can you identify zeros of polynomial functions from a graph?<br>What kinds of symmetry are found in even and odd functions? | Why are multiple types of functions needed to model real world phenomena?<br>How does visualizing a function help interpret the relationship between two variables?<br>How is the graph of an equation related to its solutions?<br>Why is it helpful to know if a function is even or odd?<br>Why is it necessary to know the symmetry and degree of a polynomial to graph it? |
| Mathematicians interpret the average rate of change of a function over a specified interval to investigate the rate at which one quantity changes with respect to another quantity. (MA10-GR.HS-S.2-GLE.1-EO.b.iii) | How can you investigate the average rate of when a function is presented graphically, symbolically or as a table?<br>What is the relationship between an average rate of change of any function and the slope of a linear function? | Why is the average rate of change important when investigating a function?<br>Why do some functions require average rates of change to be investigating over a specified interval versus the entire function? |
| Systems of non-linear functions create solutions set more complex than those of systems of linear functions. (MA10-GR.HS-S.2-GLE.4-EO.e.ii) | How can you determine from a graph or table of values the solutions to a system of equations?<br>When solving a system of non-linear equations, how many solutions could exist? | Why is it often necessary to approximate solutions to non-linear systems of equations using a table or graph? |
| Inverse functions facilitate the efficient computation of inputs of the original function. (MA10-GR.HS-S.2-GLE.1-EO.e.iii) | What is the relationship of the graph of an its inverse?<br>When is it necessary to limit the domain of a inverse function? | How do inverses functions expand our understanding of an original function?<br>Why are inverses important in mathematical modeling? |
| Logarithms, the inverse of exponential functions, provide a mechanism for transforming and solving exponential functions. (MA10-GR.HS-S.2-GLE.2-EO.a.iv) | How can you use the properties of exponents to represent an exponential function as a logarithm? | How are logarithms used to solve exponential functions?<br>Why are logarithms inverses of exponential functions? (MA10-GR.HS-S.2-GLE.1-IQ.3) |

| **Key Knowledge and Skills:** **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* |
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- Define appropriate quantities for the purpose of descriptive modeling. (MA10-GR.HS-S.1-GLE.2-EO.a.ii)
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (MA10-GR.HS-S.2-GLE.4-EO.a.ii)
- Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations and include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (MA10-GR.HS-S.2-GLE.4-EO.e.ii)
- Interpret key features of graphs and tables, for a polynomial, logarithmic, or trigonometric function, in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (MA10-GR.HS-S.2-GLE.1-EO.b.i)
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (MA10-GR.HS-S.2-GLE.1-EO.c.iv)
- Identify the effect on exponential, polynomial, logarithmic, or trigonometric graphs of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs and experiment with cases and illustrate an explanation of the effects on the graph using technology. (MA10-GR.HS-S.2-GLE.1-EO.e.i, ii)
- Compare properties of two functions (e.g., polynomial, logarithmic, or trigonometric) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (MA10-GR.HS-S.2-GLE.1-EO.c.v.3)
- Calculate and interpret the average rate of change of polynomial, logarithmic, or trigonometric functions (presented symbolically or as a table) over a specified interval and estimate the rate of change from a graph. (MA10-GR.HS-S.2-GLE.1-EO.b.iii)
- Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (MA10-GR.HS-S.2-GLE.1-EO.e.iii)
- Express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. (MA10-GR.HS-S.2-GLE.2-EO.a.iv)
- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. (MA10-GR.HS-S.2-GLE.3-EO.b.ii)
- Fit a linear or quadratic function to the data; use functions fitted to data to solve problems in the context of the data. (MA10-GR.HS-S.3-GLE.1-EO.b.ii.1)
- Informally assess the fit of a function by plotting and analyzing residuals. (MA10-GR.HS-S.2-GLE.1-EO.b.ii.2)
- Find inverse functions by solving an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (MA10-GR.HS-S.2-GLE.1-EO.e.iii)

<table>
<tr><td colspan="3">Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.<br>EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: “Mark Twain exposes the hypocrisy of slavery through the use of satire.”</td></tr>
<tr><td colspan="2">A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):</td><td>A graph of a function provides a visualization of key features of the function such as the domain, range, maxima, minima, intercepts, symmetry, end behavior and average rate of change.</td></tr>
<tr><td>Academic Vocabulary:</td><td colspan="2">define, create, explain, intersection, find, approximate, interpret, description, relationship, express, formula, sketch, graphs, tables</td></tr>
<tr><td>Technical Vocabulary:</td><td colspan="2">functions, nonlinear, model, visualization, key features, domain, range, maxima, minima, intercepts, symmetry, end behavior, average rate of change, rate, systems of nonlinear functions, solutions, linear functions, logarithms, exponential, inverse, odd, even, polynomial, logarithmic, trigonometric, geometric series, derive, rational, absolute value, base</td></tr>
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