Colorado Academic Standards

Mathematics

Preschool – Second Grade

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Colorado Council of Teachers of Mathematics

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Academy District 20

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K-12 Mathematics Coordinator
St. Vrain Valley School District

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Colorado Education Association

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Aurora Public Schools

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K-12 Mathematics Specialist
Colorado Springs School District 11

T. Vail Shoultz-McCole
Early Childhood Program Director
Colorado Mesa University

Ann Summers
K-12 Mathematics and Intervention Specialist
Littleton Public Schools

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards (CCSSI, 2010). Across the curriculum at every grade, students are expected to consistently have opportunities to engage in each of the eight practices. The practices aligned with each Grade Level Expectation in the Colorado Academic Standards represent the *strongest potential* alignments between content and the practices, and are not meant to exclude students from engaging in the rest of the practices.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Math Practice MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Practice MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

References

Common Core State Standards Initiative. (2010). *Standards for mathematical practice*.
http://www.corestandards.org/Math/Practice

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. (For more on modeling, see Appendix: Modeling Cycle.)

How to Read the Colorado Academic Standards

CONTENT AREA
Grade Level, Standard Category

COLORADO
Department of Education

Prepared Graduates:
The *PG Statements* represent concepts and skills that all students who complete the Colorado education system must master to ensure their success in postsecondary and workforce settings.

Grade Level Expectation:
The *GLEs* are an articulation of the concepts and skills for a grade, grade band, or range that students must master to ensure their progress toward becoming a prepared graduate.

Evidence Outcomes

The *EOs* describe the evidence that demonstrates that a student is meeting the GLE at a mastery level.

Academic Context and Connections

The *ACCs* provide context for interpreting, connecting, and applying the content and skills of the GLE. This includes the *Colorado Essential Skills*, which are the critical skills needed to prepare students to successfully enter the workforce or educational opportunities beyond high school embedded within statute (C.R.S. 22-7-1005) and identified by the Colorado Workforce Development Committee.

The ACCs contain information unique to each content area. Content-specific elements of the ACCs are described below.

Colorado Academic Standards
Content Area

Grade Level, Standard Category 2020 Colorado Academic Standards GLE Code

Academic Context and Connections in Mathematics:

Colorado Essential Skills and Mathematical Practices: These statements describe how the learning of the content and skills described by the GLE and EOs connects to and supports the development of the *Colorado Essential Skills* and *Standards for Mathematical Practice* named in the parentheses.

Inquiry Questions: The sample question that are intended to promote deeper thinking, reflection, and refined understandings precisely related to the GLE.

Coherence Connections: These statements relate how the GLE relates to content within and across grade levels. The first statement indicates if a GLE is *major*, *supporting*, or *additional* work of the grade. Between 65% and 85% of the work of each grade (with P-2 at the high end of that range) should be focused on the GLEs labeled as **major work**. The remainder of the time should focus on **supporting work** and **additional work**, where it can appropriately support and compliment students' engagement in major work. **Advanced outcomes**, marked with a (+), represent content best saved for upper-level math courses in a student's final three semesters of high school. The remaining statements describe how the GLE and EOs build from content learned in prior grades, connects to content in the same grade, and supports learning in later grades.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Preschool Learning and Development Expectation:

P.CC.A. Counting & Cardinality: Know number names and the count sequence.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Count verbally or sign to at least 20 by ones.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Count and use numbers as they play with children.
2. Take advantage of every opportunity to count with children in a practical and authentic setting.

Examples of Learning/Children May:

1. Read stories, sing songs, and act out poems and finger plays that involve counting, numerals, and shapes.
2. Practice saying a sequence of number words.
3. Respond to the question, "What comes after four?" with "One, two, three, four … five!"

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 24–36 months, children say or sign some number words in sequence, starting with one, and understand that counting words are separate words, such as "one," "two," "three," versus "onetwothree."
3. In preschool, learning the counting sequence is part of learning progressions that go (a) from saying the counting words to counting out objects and (b) from speaking number words to writing base-ten numerals.
4. In kindergarten, students count to 100 by ones and tens and count forward from a given number.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

Preschool Learning and Development Expectation:

P.CC.B. Counting & Cardinality: Recognize the number of objects in a small set.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

2. Instantly recognize, without counting, small quantities of up to five objects and say or sign the number.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Hold five or fewer objects in a closed hand, then open it briefly for the child, close it again, and ask, “How many did you see?”
2. Quickly show children a card with five or fewer dots, then hide it and ask who can say how many dots they saw.
3. Ask children to place their hands where they can’t see them, then show a small number on their fingers, then have the children check their work by looking at their hands.

Examples of Learning/Children May:

1. Play with a friend and say without counting, “I have five big rocks and you have five little rocks. We have the same.”
2. Find fewer objects or objects in patterns (like two rows of 2 to make four) easier to subitize.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children develop an understanding of what whole numbers mean and become increasingly able to quickly recognize the number of objects in a small set (known as subitizing).
3. In preschool, subitizing facilitates efficient counting.
4. In kindergarten, students count to determine the number of up to 20 arranged or up to 10 scattered objects.

Prepared Graduates:

MP6. Attend to precision.

Preschool Learning and Development Expectation:

P.CC.C. Counting & Cardinality: Understand the relationship between numbers and quantities.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

3. Say or sign the number names in order when counting, pairing one number word that corresponds with one object, up to at least 10.
4. Use the number name of the last object counted to answer "How many?" questions for up to approximately 10 objects.
5. Accurately count as many as five objects in a scattered configuration or out of a collection of more than five objects.
6. Understand that each successive number name refers to a quantity that is one larger.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Play age-appropriate games that involve counting spaces or objects.
2. Count to five from thumb to pinky on an open hand, then close the hand except for the pinky and ask, "How many fingers are still showing?" to see if a child answers one or five.
3. Help children count by pointing to objects or drawings of objects, then confirming the total by asking, "So how many are there altogether?"
4. Provide opportunities to count objects for lunch, such as plates, napkins, and cups.

Examples of Learning/Children May:

1. Match a group of 1 to 10 objects with written and spoken numbers.
2. Play simple games that match numbers to a movement of spaces on a game board.
3. Take a specified number of crackers from a bowl during snack time.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children coordinate verbal counting with objects by pointing at each object for each number word (known as one-to-one correspondence) and develop an understanding that the last number in the sequence represents how many in the group (known as cardinality).
3. In preschool, students connect the process of counting to a conceptual understanding of cardinality.
4. In kindergarten, students count to determine the number of objects using one-to-one correspondence and cardinality for up to 20 objects in a line or 10 scattered objects.

Prepared Graduates:

MP7. Look for and make use of structure.

Preschool Learning and Development Expectation:

P.CC.D. Counting & Cardinality: Compare numbers.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

7. Identify whether the number of objects in one group is more than, less than or the same as objects in another group for up to at least five objects.
8. Identify and use numbers related to order or position from first to fifth.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Have children group and order materials when cleaning up.
2. Describe quantities using vocabulary including more than, less than, and equal to.
3. Provide opportunities for children to count, group, and order objects and materials.
4. Put four counting chips inside a circle and one chip outside the circle, then ask, "Which has more, inside or outside? Which has fewer chips?"

Examples of Learning/Children May:

1. Count, group, and sort objects and materials.
2. Be able to express a preference for greater numbers of things (such as candy or toys) when comparing groups of different sizes.
3. Say phrases like, "There are more cookies in this box," or "There are fewer pencils on that table than on this one."
4. Identify which item is first, second, third, etc., when pointing to items or talking about events that are ordered.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children begin to count and compare same-size objects (with adult assistance) and begin to understand that the number of objects is independent of the size of the objects.
3. In kindergarten, students identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for up to 10 objects. Students also compare two numbers between 1 and 10 presented as written numerals.

Prepared Graduates:

MP5. Use appropriate tools strategically.

Preschool Learning and Development Expectation:

P.CC.E. Counting & Cardinality: Associate a quantity with written numerals up to 5 and begin to write numbers.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

9. Associate a number of objects with a written numeral 0–5.
10. Recognize and, with support, write some numerals up to 10.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Play games with children where spinning a wheel with numbers or the number written on a card is associated with the need to count that number of objects or spaces.
2. Help a child write or trace using any writing tool the numeral corresponding to his or her age.
3. Support the use of a numeral by connecting it to a group of objects or a picture of objects to help students associate the numeral to a quantity.

Examples of Learning/Children May:

1. Match a group of 1 to 5 objects with written and spoken numbers.
2. Copy a printed numeral using their own handwriting.
3. Play games that involve matching numerals to numbers of objects, such as dots on cards.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children develop an understanding that a written numeral represents a quantity and uses symbols, like tally marks, to represent numerals.
3. In preschool, work with numerals is still in its early stages. Writing numerals does not become a focus until kindergarten, but it can be done in preschool to support other work in mathematics and writing.
4. In kindergarten, students write numbers from 0 to 20 and associate a number of objects with the written numerals 0–20.

Prepared Graduates:

MP4. Model with mathematics.

Preschool Learning and Development Expectation:

P.OA.A. Operations & Algebraic Thinking: Understand addition as adding to and understand subtraction as taking away from.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Represent addition and subtraction in different ways, such as with fingers, objects, and drawings.
2. Solve addition and subtraction problems set in simple contexts. Add and subtract up to at least five to or from a given number to find a sum or difference up to 10.
3. With adult assistance, begin to use counting on (adding 1 or 2, for example) from the larger number for addition.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Use fingers on both hands to represent addition.
2. Ask a child with five crackers, "If you eat three of your crackers, how many will you have left?"
3. Ask "How many more?" questions, such as, "We have three children in this group. How many more children do we need to make a group of five?"

Examples of Learning/Children May:

1. Add a group of three and a group of two, counting "One, two three …" and then counting on "Four, five!" while keeping track using their fingers.
2. Take three away from five, counting "Five, four, three … two!" while keeping track using their fingers.
3. Say after receiving more crackers at snack time, "I had two and now I have four."
4. Predict what will happen when one more object is taken away from a group of five or fewer objects, and then verify their prediction by taking the object away and counting the remaining objects.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children develop beginning understandings of adding and subtracting with the help of objects and adult support.
3. In preschool, students should work with small numbers and simpler problem subtypes (see Appendix, Table 1).
4. In kindergarten, students add and subtract within 10 using objects or drawings to represent problems and fluently add and subtract within 5.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Preschool Learning and Development Expectation:

P.OA.B. Operations & Algebraic Thinking: Understand simple patterns.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

4. Fill in missing elements of simple patterns.
5. Duplicate simple patterns in a different location than demonstrated, such as making the same alternating color pattern with blocks at a table that was demonstrated on the rug. Extend patterns, such as making an eight-block tower of the same pattern that was demonstrated with four blocks.
6. Identify the core unit of sequentially repeating patterns, such as color in a sequence of alternating red and blue blocks.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Provide everyday opportunities to explore numbers and patterns, such as setting the table with a cup, plate, and fork for each person.
2. Provide opportunities to observe naturally occurring patterns within the indoor and outdoor environments, such as looking at patterns in the bricks of a building or patterns in art and design.
3. Introduce songs and movement patterns where children can extend and grow the pattern.

Examples of Learning/Children May:

1. Use art materials and other objects to create or replicate patterns (e.g., weaving, stringing beads, stacking blocks, or drawing repeating pictures).
2. Recognize patterns in a story or song.
3. Identify two blocks, one red and one blue, as the core unit of a longer pattern using alternating red and blue blocks.
4. Sequence story cards to show beginning, middle, and end.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children recognize and work with simple patterns (like ABAB) in different forms, such as patterns of objects, numbers, sounds, and movements.
3. In preschool, students may recognize and duplicate more complicated patterns, such as ABC, ABB, and AABB.
4. In kindergarten, pattern recognition is embedded in and focused on early numeracy, such as counting by tens, number composition/decomposition, making tens, describing attributes of objects, and classifying objects into categories.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

Preschool Learning and Development Expectation:

P.MD.A. Measurement & Data: Measure objects by their various attributes using standard and nonstandard measurement and use differences in attributes to make comparisons.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Use comparative language, such as shortest, heavier, biggest, or later.
2. Compare or order up to five objects based on their measurable attributes, such as height or weight.
3. Measure using the same unit, such as putting together snap cubes to see how tall a book is.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Follow a pictorial recipe and let children measure, pour, and stir the ingredients while asking questions like, “How many cups of flour does the recipe show we need to put in the bowl?”
2. Provide opportunities for children to sort, classify and group household objects and materials.
3. Ask questions of measurement (e.g., “How many steps does it take to walk from the front door to your cubby?” or “How many blocks long is your arm?”).
4. Offer a variety of measuring tools and models, such as rulers, yardsticks, measuring tapes, measuring cups, scales, and thermometers. (Children may not use each of these correctly, but they are developing early understandings of how tools measure things.)
5. Provide opportunities for children to use non-standard measuring tools such as cubes, paperclips, blocks, etc.

Examples of Learning/Children May:

1. Sort objects by physical characteristics such as a color or size.
2. Group objects according to their size, using standard and nonstandard forms of measurement (e.g., height, weight, length, color, or brightness).
3. Explore various processes and units for measurement and begin to notice different results of one method or another.

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, children develop an understanding that attributes can be described and compared in simple ways, such as one child being taller than another.
3. In preschool, this expectation connects with counting, comparing numbers, and comparing shapes.
4. In kindergarten, students describe multiple measurable attributes of an object and make direct comparisons of two objects with a measurable attribute in common.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

Preschool Learning and Development Expectation:

P.G.A. Geometry: Identify, describe, compare, and compose shapes.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Name and describe shapes in terms of length of sides, number of sides, and number of angles/corners.
2. Correctly name basic shapes (circle, square, rectangle, triangle) regardless of size and orientation.
3. Analyze, compare, and sort two-and three-dimensional shapes and objects in different sizes. Describe their similarities, differences, and other attributes, such as size and shape.
4. Compose simple shapes to form larger shapes.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Use a sensory table with various bowls, cups, or other containers to encourage activities with shapes and sorting.
2. Provide children with puzzles made of simple geometric shapes and encourage saying the names of shapes as they play.
3. Discuss geometric shapes in terms of their attributes, such as “This is a circle. It’s perfectly round with no bumps or corners. This is a triangle. It has three sides and three angles.”
4. Use a variety of lengths and angles in their shapes (such as scalene triangles, long and thin rectangles) as well as more common configurations of shapes (such as equilateral triangles).

Examples of Learning/Children May:

1. Match, sort, group, and name basic shapes found outside or in the classroom.
2. Use pattern tiles to make shapes out of other shapes, such as putting two squares side-by-side to make a non-square rectangle.
3. Put away blocks and/or tiles into different containers based on the number or length of sides.

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, children start by recognizing circles and squares and then add triangles and other shapes. As understanding of shape develops, children identify sides and angles as distinct parts of shapes.
3. In preschool, this expectation connects with measuring and comparing objects by their attributes.
4. In kindergarten, students identify and describe squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

Prepared Graduates:

MP6. Attend to precision.

Preschool Learning and Development Expectation:

P.G.B. Geometry: Explore the positions of objects in space.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

5. Understand and use language related to directionality, order, and the position of objects, including up/down and in front/behind.
6. Correctly follow directions involving their own position in space, such as "Stand up" and "Move forward."

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Provide opportunities for conversation using everyday words to indicate space location, shape, and size of objects, saying things like, "You crawled under the picnic table, over the tree stump, and now you are in the tunnel slide!"
2. Help children organize toys, pointing out concepts such as "in," "on," and "beside."

Examples of Learning/Children May:

1. Use the vocabulary of geometry and position to describe shapes within the room and surrounding environment.
2. Understand relational directions, such as "Please put a mat under each plate."

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, students develop spatial vocabulary and become able to follow directions involving their own position in space.
3. In preschool and early elementary, students work with shapes and their attributes in increasingly sophisticated ways over time.
4. In kindergarten, students describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, in front of, behind, and next to.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.CC.A. Counting & Cardinality: Use number names and the count sequence.

Evidence Outcomes

Students Can:

1. Count to 100 by ones and by tens. (CCSS: K.CC.A.1)
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). (CCSS: K.CC.A.2)
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). (CCSS: K.CC.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the number sequence from 1 to 9 repeats between the decade numbers, except in the spoken numbers between 10 and 20. (MP7)
2. Reason that counting to 100 by tens reaches the same number as can be counted repeatedly by ones. (MP8)

Inquiry Questions:

1. When might you want to count by tens instead of ones?
2. When might you want to start counting from a number other than one?
3. What number can we use to show we have nothing to count?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students understand that number words have a sequence and that the words are separate (not “onetwothree”).
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students extend the counting sequence to 120.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

K.CC.B. Counting & Cardinality: Count to determine the number of objects.

Evidence Outcomes

Students Can:

4. Apply the relationship between numbers and quantities and connect counting to cardinality. (CCSS: K.CC.B.4)
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.B.4.a)
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.B.4.b)
   c. Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.B.4.c)
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. (CCSS: K.CC.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Progress from thinking about numbers as the result of the process of counting to abstractly thinking about numbers as mental objects of their own—especially the quantity 10. (MP2)
2. Explain how the number reached when counting on is a relationship between the quantity started from and the quantity added. (MP3)
3. Make counting efficient by following rows, columns, or other patterns in a group of arranged objects. (MP7)

Inquiry Questions:

1. How is counting to five different from the number five?
2. What number is one larger than four? What number is one larger than seven?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build conceptions of what whole numbers mean, of subitizing, of one-to-one correspondence between verbal counting and objects, and of cardinality.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten understanding.
4. In Grade 1, students use their understanding of counting and cardinality to add and subtract within 20.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

K.CC.C. Counting & Cardinality: Compare numbers.

Evidence Outcomes

Students Can:

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to 10 objects.) (CCSS: K.CC.C.6)
7. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make reasoned arguments about the relative sizes of groups, such as by matching objects of two groups and seeing which has extra objects, or by counting the objects in each group and seeing which has the number further in the counting sequence. (MP3)
2. Use precise language to describe why one quantity is less than, greater than, or equal to another, and avoid mixing and misusing different ways of quantifying such as dimension, weight, or magnitude. (MP6)

Inquiry Questions:

1. Other than counting, how might you decide whether one set has more objects than another?
2. Which is more, 3 small cookies or 2 big cookies? What makes this difficult to answer?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build an understanding of same versus different numbers of items, numbers of objects versus their size, and ordering from first to fifth.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from counting to counting on and (b) from comparison by matching to comparison by numbers to comparison involving adding and subtracting.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers. Students also organize data into categories and compare how many more or less are in one category than in another.

Prepared Graduates:

MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.NBT.A. Number & Operations in Base Ten: Work with numbers 11–19 to gain foundations for place value.

Evidence Outcomes

Students Can:

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be precise in drawings, diagrams, and numerical recordings about objects or symbols that represent ones and objects or symbols that represent tens. (MP6)
2. See the structure of a number as composed of its base-ten units. (MP7)
3. Repeat the reasoning afforded by the uniformity of the base-ten system, where 10 copies compose 1 base-ten unit of the next highest value. (MP8)

Inquiry Questions:

1. Can you show the number 13 as ten ones and some more ones? How many more ones than tens are there?
2. In the number 11, what makes the “1” on the left different from the “1” on the right? Could you show this with objects or a diagram?
3. What would a number called “ten four” look like? What word do we usually say for this number?
4. Why might someone call the number 17 “ten seven?”

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students develop conceptions of addition and subtraction when adding to and taking away from small collections of objects.
3. In kindergarten, this expectation is part of a progression from comparison by spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers.

Prepared Graduates:

MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

K.OA.A. Operations & Algebraic Thinking: Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.

Evidence Outcomes

Students Can:

1. Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.A.1)
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.A.2)
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). (CCSS: K.OA.A.3)
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.A.4)
5. Fluently add and subtract within 5. (CCSS: K.OA.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of real-world situations involving addition and subtraction (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. (MP4)
3. Act out adding and subtracting situations by representing quantities in the situation with objects, fingers, and math drawings. (MP5)
4. Use the equal sign consistently and appropriately. (MP6)

Inquiry Questions:

1. How could you show me adding 3 and 2?
2. How could you show me 3 take away 2?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students represent addition and subtraction within 5 with fingers, objects, and drawings.
3. In kindergarten, this expectation is part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Appendix, Table 1).
4. In Grade 1, students understand properties of operations, the relationship between addition and subtraction, and add and subtract within 20.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

K.MD.A. Measurement & Data: Describe and compare measurable attributes.

Evidence Outcomes

Students Can:

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. (CCSS: K.MD.A.1)
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* (CCSS: K.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of their world by comparing and ordering objects by their attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Be precise about meanings related to size when describing an object's height, weight, or other attribute. (MP6)

Inquiry Questions:

1. What does it mean for one object to be "bigger" than another?
2. If you are standing on a chair, how should your height be measured differently than if you were standing on the floor?
3. If an object is moved, does that change its size?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students develop conceptions of measurable attributes of objects and comparisons based on those attributes.
3. In kindergarten, this expectation can contribute to students' understandings of measurable attributes, comparison, and conservation of length, all of which connect to progressions in geometry, the number system, and to future work in ratio and proportion.
4. In Grade 1, students measure lengths directly and by iterating length units, and express the length of an object as a whole number of length units.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

K.MD.B. Measurement & Data: Classify objects and count the number of objects in each category.

Evidence Outcomes

Students Can:

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.) (CCSS: K.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Group objects into categories to help make sense of problems. (MP1)
2. Abstract individual objects into new conceptual groups. (MP2)
3. Choose appropriate representations of objects and categories. (MP5)

Inquiry Questions:

1. How can numbers of objects be represented to make comparisons?
2. How can objects be categorized in different ways?
3. How can an object's attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In preschool, students use differences in attributes to make comparisons.
3. In kindergarten, this expectation supports the work of counting and comparing numbers and is part of a progression of learning how to analyze categorical data.
4. In Grade 1, students organize, represent, and interpret data with up to three categories.

Prepared Graduates:

MP4. Model with mathematics.

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.A. Geometry: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Evidence Outcomes

Students Can:

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.A.1)
2. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.A.2)
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). (CCSS: K.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe the physical world from geometric perspectives, e.g., shape, orientation, and spatial relationships. (MP4)
2. Reflect an increasing understanding of shapes by using increasingly precise language to describe them. (MP6)
3. Sort shapes into categories (squares, circles, triangles, etc.) based on attributes of the shapes. (MP7)

Inquiry Questions:

1. For a given shape, what attributes make an example of that shape different from a non-example? For example, "Why is this shape (point to a square) a square, while this shape (point to a non-square) is not?"
2. What are the ways of describing where an object is?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students learn about circles, squares, triangles, and their parts.
3. In kindergarten, this expectation connects with the work of analyzing, comparing, creating, and composing shapes.
4. In future grades, students calculate area and surface area of these and other shapes.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.B. Geometry: Analyze, compare, create, and compose shapes.

Evidence Outcomes

Students Can:

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). (CCSS: K.G.B.4)
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS: K.G.B.5)
6. Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”* (CCSS: K.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use experiences with multiple examples of a type of shape to develop a concept image (see glossary) of that shape from which they can abstract common features. (MP2)
2. Model shapes in the world by building them with components or drawing representations of them. (MP4)
3. Use patterns or structures when making comparisons or compositions of shapes. (MP7)

Inquiry Questions:

1. Can you change a shape into a different kind of shape by rotating it?
2. What kinds of pictures can you make by combining shapes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students understand and use language related to directionality, order, and the position of objects, such as up/down and in front/behind.
3. In kindergarten, this expectation connects with identifying and describing shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
4. In Grade 1, students classify, compose, and partition shapes.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.A. Number & Operations in Base Ten: Extend the counting sequence.

Evidence Outcomes

Students Can:

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make use of the base-ten counting structure when using special words at the decades, like “sixty” and “seventy.” (MP7)

Inquiry Questions:

1. When might someone want to count by tens instead of ones?
2. Which numbers can be written with two numerals and which numbers are written with three?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students count to 100 by ones and tens, count forward from a given number, and connect counting to cardinality.
3. In Grade 1, this expectation connects with understanding place value and with adding and subtracting within 20.
4. In Grade 2, students extend their place value understanding to hundreds and three-digit numbers, and use this along with the properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.B. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: (CCSS: 1.NBT.B.2)
   a. 10 can be thought of as a bundle of ten ones — called a "ten." (CCSS: 1.NBT.B.2.a)
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: 1.NBT.B.2.b)
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS: 1.NBT.B.2.c)
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. (CCSS: 1.NBT.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of quantities and their relationships in problem situations. (MP1)
2. Abstract 10 ones into a single conceptual object called a ten. (MP2)
3. Model ones and tens with objects and mathematical representations. (MP4)
4. See the structure of a number as its base-ten units. (MP7)

Inquiry Questions:

1. What does the position of a digit tell you about its value?
2. What are two ways to describe the number 30?
3. Why was a place value system developed? What might numbers look like without it?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students decompose numbers from 11 to 19 into ten ones and further ones.
3. In Grade 1, this expectation connects with extending the counting sequence and using place value understanding and properties of operations to add and subtract within 100.
4. In Grade 2, students understand hundreds and place value of three-digit numbers, and use this along with the properties of operations to add and subtract.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.C. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (CCSS: 1.NBT.C.4)
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (CCSS: 1.NBT.C.5)
6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 1.NBT.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Perform computation with addition and subtraction while making connections to the properties of operations and to place value structure. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Model quantities with drawings or equations to make sense of place value. (MP1)
3. Use the base-ten structure to add and subtract, including adding and subtracting multiples of ten. (MP7)

Inquiry Questions:

1. Can you add or subtract ten without having to count by ones?
2. How does modeling addition look different if you add tens and ones separately compared to counting on by tens then by ones?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students model and describe addition as putting together and adding to, and subtraction as taking part and taking from, using objects or drawings. Students also work with numbers 11–19 to gain foundations for place value.
3. In Grade 1, this expectation connects with understanding place value and adding and subtracting within 20.
4. In Grade 2, students understand place value for three-digit numbers and use that understanding and properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP4. Model with mathematics.

Grade Level Expectation:

1.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.1)
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of problems by relating objects, drawings, and equations. (MP1)
2. Use cubes, number racks, ten frames and other models to represent addition and subtraction situations in real-world contexts. (MP4)

Inquiry Questions:

1. How can you use cubes to help you compare two numbers?
2. (Given a representation of a value less than ten) How many more do you need to make ten?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students add and subtract within 10 by using objects or drawings to represent problems.
3. In Grade 1, this expectation connects with comparing, adding, and subtracting numbers, including measurement and data activities.
4. In Grade 2, students represent and solve real-world problems involving addition and subtraction within 100, with fluency expected within 20.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP7. Look for and make use of structure.

Grade Level Expectation:

1.OA.B. Operations & Algebraic Thinking: Understand and apply properties of operations and the relationship between addition and subtraction.

Evidence Outcomes

Students Can:

3. Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) *Examples: If* $8 + 3 = 11$ *is known, then* $3 + 8 = 11$ *is also known. (Commutative property of addition.) To add* $2 + 6 + 4$*, the second two numbers can be added to make a ten, so* $2 + 6 + 4 = 2 + 10 = 12$. *(Associative property of addition.)* (CCSS: 1.OA.B.3)
4. Understand subtraction as an unknown-addend problem. *For example, subtract* $10 - 8$ *by finding the number that makes* 10 *when added to* 8. (CCSS: 1.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of addition and subtraction by applying properties of operations and working with different problem types (see Appendix, Table 1). (MP1)
2. Use properties of operations to recognize equivalent forms of equations. (MP7)

Inquiry Questions:

1. How could you explain why $3 + 8$ and $8 + 3$ both equal 11?
2. How can you use the number line to show how you might use adding OR subtracting to solve the same problem?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.
3. In Grade 1, this expectation connects with representing and solving problems involving addition and subtraction and with adding and subtracting within 20.
4. In future grades, students use place value understanding and properties of operations to add and subtract within larger number ranges, then to perform multi-digit arithmetic. Later, students use these concepts to build fractions from unit fractions, and to apply and extend their understandings of arithmetic to algebraic expressions.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

1.OA.C. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (CCSS: 1.OA.C.5)
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). (CCSS: 1.OA.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use multiple strategies to think about problems and see how the quantities involved support the use of some strategies over others. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make use of the structure of numbers when making tens or when creating equivalent but easier or known sums. (MP7)

Inquiry Questions:

1. Which would you prefer when adding $4 + 7$: starting with 7 and counting up 4 or starting with 4 and counting up 7? Why?
2. Why does knowing doubles like $4 + 4$ or $5 + 5$ help when adding $4 + 5$?
3. How does counting on to add and subtract within 20 make it easier to use fingers even though we have only 10 fingers?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students understand the relationship between numbers and quantities and connect counting to cardinality.
3. In Grade 1, this expectation connects with place value understanding, properties of addition and subtraction, the relationship between addition and subtraction, and with representing and solving problems involving addition and subtraction.
4. In Grade 2, students fluently add and subtract within 20 using mental strategies and know from memory all sums of two one-digit numbers.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

1.OA.D. Operations & Algebraic Thinking: Work with addition and subtraction equations.

Evidence Outcomes

Students Can:

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?* $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$. (CCSS: 1.OA.D.7)
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 + ? = 11$, $5 = \_ - 3$, $6 + 6 = \_$. (CCSS: 1.OA.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of quantities and their relationships in problem situations. (MP2)
2. Question assumptions about the meaning of the equals sign and construct viable arguments. (MP3)

Inquiry Questions:

1. What does it mean for two sides of an equation to be “equal”? How can $2 +$ 3 “equal” 5?
2. (Given $4 = 4$ If you add 2 more to the 4 on the right, how many do you need to add on the left to make a true statement? How would you write that as an equation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students represent addition and subtraction with equations without needing to understand the meaning of the equal sign.
3. In Grade 1, this expectation connects with representing and solving problems involving addition and subtraction.
4. In Grade 2, students work with equal groups of objects to gain foundations for multiplication. In Grade 4, students build fractions from unit fractions and apply addition and subtraction to concepts of angle and angle measurement.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

1.MD.A. Measurement & Data: Measure lengths indirectly and by iterating length units.

Evidence Outcomes

Students Can:

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS: 1.MD.A.1)
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* (CCSS: 1.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Abstract comparisons between lengths using statements like $A > B$. (MP2)
2. Use the transitive property to explain if A is longer than B, and B is longer than C, then A must be longer than C. (MP3)
3. Devise different ways to represent the same data set and discuss the strengths and weaknesses of each representation. (MP5)
4. Consider the endpoints of objects when measuring and making comparisons. (MP6)

Inquiry Questions:

1. How is it possible for 5 sticks placed end-to-end to be equal in length to 6 sticks placed end-to-end?
2. Which is longer, the total length of two sticks placed end-to-end vertically or the same two sticks placed end-to-end horizontally?
3. What objects in this classroom are the same length as (or longer than, or shorter than) your forearm?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students directly compare two objects with a measurable attribute in common.
3. In Grade 1, this expectation is part of a progression of learning that develops conceptions of comparison, conservation, seriation, and iteration.
4. In Grade 2, students measure and estimate lengths in standard units.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

1.MD.B. Measurement & Data: Tell and write time.

Evidence Outcomes

Students Can:

3. Tell and write time in hours and half-hours using analog and digital clocks. (CCSS: 1.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Tell and manage time to be both personally responsible and responsible to the needs of others. (Personal Skills: Personal Responsibility)
2. Recognize that time is a quantity that can be measured with different degrees of precision. (MP6)

Inquiry Questions:

1. How long is two half-hours?
2. If the time is 2:30, where would the minute hand be pointing on an analog clock?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In kindergarten, students are not expected to learn how to tell and write time.
3. In Grade 2, students tell and write time from analog and digital clocks to the nearest five minutes.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

1.MD.C. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (CCSS: 1.MD.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Ask and answer questions about categorical data based on representations of the data. (MP1)
2. Group similar individual objects together and abstract those objects into a new conceptual group. (MP2)
3. Devise different ways to display the same data set then discuss relative strengths and weaknesses of each scheme. (MP5)
4. Use appropriate labels and units of measure. (MP6)

Inquiry Questions:

1. How do different representations of data indicate there are more objects in one category than in another category?
2. How can objects be categorized in different ways?
3. How can an object's attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In kindergarten, students classify objects into given categories, count the numbers of objects in each category, and sort the categories by count.
3. In Grade 1, this expectation supports representing and solving problems involving addition and subtraction, which is major work of the grade.
4. In Grade 2, students draw a picture graph and a bar graph to represent a data set with up to four categories, and solve put-together, take-apart, and compare problems using the information in a bar graph.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. (CCSS: 1.G.A.1)
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names, such as "right rectangular prisms.") (CCSS: 1.G.A.2)
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS: 1.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Demonstrate flexibility, imagination, and inventiveness in composing two-dimensional and three-dimensional shapes to create composite shapes. (Entrepreneurial Skills: Informed Risk Taking)
2. Sort, classify, build, or draw shapes in terms of defining attributes versus non-defining attributes. (MP1)
3. Determine how to partition a given circle or rectangle into two and four equal shares and describe the whole in terms of equal shares. (MP2)
4. Justify whether a shape belongs in a given category by differentiating between defining attributes and non-defining attributes. (MP3)
5. Analyze how composite shapes can be formed by, or decomposed into, basic shapes. (MP7)

Inquiry Questions:

1. Which properties of shapes are most important when you decide if a shape belongs in a group with other shapes?
2. What kinds of objects can you find in your school or home that are made up of two or more different shapes being put together?
3. In how many different ways can you create two or four equal shares in a rectangle?

Coherence Connections:

1. This expectation is an addition to the major work of the grade.
2. In kindergarten, students identify, describe, analyze, compare, create, and compose shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
3. In Grade 2, students recognize and draw shapes having specified attributes and partition circles and rectangles into two, three, or four equal shares. In Grade 3, students develop understanding of fractions as numbers.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.A. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.A.1)
   a. 100 can be thought of as a bundle of ten tens — called a “hundred.” (CCSS: 2.NBT.A.1.a)
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.A.1.b)
2. Count within 1000; skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.A.2)
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.A.3)
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Abstract 10 ones into a single conceptual object called a ten and abstract 100 ones or 10 tens into a single conceptual object called a hundred. (MP2)
2. Compose, decompose, and compare three-digit numbers according to their base-ten structure. (MP7)

Inquiry Questions:

1. How many hundreds are in the number “four hundred five”? How do you know? How many tens are in the number "four hundred five? How do you know?
2. How many times do you need to skip count by 5s to count as far as skip counting by 10s once?
3. How many times do you need to skip count by 10s to count as far as skip counting by 100 once?
4. Why is any two-digit number that starts with 5 always larger than a two-digit number that starts with 3?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students understand place value for two-digit numbers.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract and with working with equal groups of objects to gain foundations for multiplication.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.B.5)
6. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.B.6)
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.B.7)
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.B.8)
9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) (CCSS: 2.NBT.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Relate concrete or mental strategies for adding and subtracting within 100 to a written method. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of place value by modeling quantities with drawings or equations. (MP1)
3. Use the base-ten structure to add and subtract, composing and decomposing ones, tens, and hundreds as necessary. (MP7)

Inquiry Questions:

1. Why might it be helpful to view subtraction as an unknown addend problem? (e.g., $278 + ? = 425$)
2. How might you rewrite $38 + 47 + 93 + 62$ to make it easier to solve? How do you know it is OK to rewrite it?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students use place value and properties of operations to make sense of the relationship between addition and subtraction.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and fluently adding and subtracting within 20.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic, including fluently adding and subtracting within 1000.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

2.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 1) (CCSS: 2.OA.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Decontextualize word problems, use mathematics to solve, and then recontextualize to provide the answer in context. (MP2)
2. Represent situations in word problems using drawings and equations with symbols for unknown numbers. (MP4)

Inquiry Questions:

1. (Given a word problem) What is the unknown quantity in this problem?
2. (Given an addition or subtraction problem) How might you use a model to represent this problem?
3. Does the word “more” in a word problem always mean that you will use addition to solve the problem? Why or why not?

Coherence Connections:

1. This expectation represents the major work of the grade.
2. In Grade 1, students use place value understanding and properties of operations to represent and solve problems involving addition and subtraction.
3. This expectation connects with other ideas in Grade 2: (a) using place value understanding and properties of operations to add and subtract, (b) relating addition and subtraction to length, (c) working with time and money, and (d) representing and interpreting data.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

Grade Level Expectation:

2.OA.B. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

2. Fluently add and subtract within 20 using mental strategies. (See 1.OA.C.6 for a list of strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. (CCSS: 2.OA.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize those problems that can be solved mentally versus those that require the use of objects, diagrams, or equations. (MP5)
2. Add and subtract within 20 quickly, accurately, and flexibly. (MP6)

Inquiry Questions:

1. How can you use addition and subtraction facts you know to quickly determine facts that you don't know?
2. Why do you think it is important to know your addition and subtraction facts?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students use objects and drawings to add and subtract within 20 in preparation for fluency with mental strategies in Grade 2.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

2.OA.C. Operations & Algebraic Thinking: Work with equal groups of objects to gain foundations for multiplication.

Evidence Outcomes

Students Can:

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.C.3)
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore the arrangement of objects and how some arrangements afford mathematical power to solve problems. (Entrepreneurial Skills: Creativity/Innovation)
2. Reason about what it means for numbers to be even and odd. (MP2)
3. Explain why a group of objects is even or odd and if a strategy for deciding works with any group of objects. (MP3)

Inquiry Questions:

1. What does it mean for a number to be even?
2. Do two equal addends always result in an even sum? Why or why not?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students work with addition and subtraction equations.
3. In Grade 2, this expectation connects with understanding place value for three-digit numbers.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.A. Measurement & Data: Measure and estimate lengths in standard units.

Evidence Outcomes

Students Can:

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.A.1)
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.A.2)
3. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.A.3)
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Consider the correctness of another students' measurement in which they lined up three large and four small blocks and claimed a path was "seven blocks long." (MP3)
2. Choose between different measurement tools depending on the objects they need to measure. (MP5)
3. Determine when it is appropriate to estimate an object's length or when a more precise measurement is needed. (MP6)

Inquiry Questions:

1. What do the numbers on a ruler represent?
2. What is the more appropriate tool for measuring the length of your school hallway, a 1-foot ruler or a 25-foot measuring tape?
3. When is it appropriate to estimate length? When is it not appropriate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students measure lengths indirectly and by iterating length units.
3. In Grade 2, this expectation connects with relating addition and subtraction to length and with representing and interpreting data.
4. In Grade 3, students (a) develop understanding of fractions as numbers, (b) solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (c) use concepts of area and relate area to multiplication and to addition.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

2.MD.B. Measurement & Data: Relate addition and subtraction to length.

Evidence Outcomes

Students Can:

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.B.5)
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize problems involving lengths and identify possible solutions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Build on experiences with measurement tools to understand number lines as a more abstract tool for working with quantities. (MP2)
3. Use mathematical representations, like drawings and equations, to model scenarios described in word problems. (MP4)

Inquiry Questions:

1. When might it be necessary to measure parts of objects and then combine those parts together?
2. How is a number line like a ruler?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students add and subtract within 20 and express the length of an object as a whole number of length units.
3. In Grade 2, this expectation connects with measuring and estimating lengths in standard units and with representing and interpreting data.
4. In Grade 3, students develop an understanding of a fraction as a number on a number line.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.C. Measurement & Data: Work with time and money.

Evidence Outcomes

Students Can:

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.C.7)
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have two dimes and three pennies, how many cents do you have?* (CCSS: 2.MD.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Tell and manage time to be both personally responsible and responsible to the needs of others. (Personal Skills: Personal Responsibility)
2. Make sense of word problems involving money. (MP1)
3. Recognize that time is a quantity that can be measured with different degrees of precision. (MP6)

Inquiry Questions:

1. If the time is 2:25, where would the minute hand be pointing on an analog clock?
2. Does the size of a coin indicate the value of the coin?
3. How is money like our base-ten number system, where it takes ten of one unit to make the next unit (ten ones makes a ten, ten tens make a hundred)? In what ways is it different?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students tell and write time in hours and half-hours using analog and digital clocks.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction.
4. In Grade 3, students tell and write time to the nearest minute and measure time intervals in minutes.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

2.MD.D. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.D.9)
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (see Appendix, Table 1) using information presented in a bar graph. (CCSS: 2.MD.D.10)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Organize objects according to measures or categories to help make sense of problems. (MP1)
2. Organize measurement and categorical data into categories based on size or type so comparisons can be made between categories instead of between individual objects. (MP2)
3. Discuss ways in which bar graph orientation (horizontal or vertical), order, thickness, spacing, shading, colors, etc. make the graphs easier or more difficult to interpret. (MP5)

Inquiry Questions:

1. How is organizing objects by length measurements, rounded to the nearest unit, similar to and different from organizing objects by categories?
2. (Given a bar graph representation of up to four categories of animals) How many more birds are there than hippos? How many more giraffes would there need to be in order for the number of giraffes to equal the number of elephants?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students organize, represent, and interpret data with up to three categories and compare how many more or less are in one category than another.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and with relating addition and subtraction to length.
4. In Grade 3, students draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

2.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.A.1)
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.A.2)
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Demonstrate flexibility, imagination, and inventiveness in drawing shapes having specified attributes and in partitioning circles and rectangles into equal shares. (Entrepreneurial Skills: Informed Risk Taking)
2. Explore various ways of partitioning shapes into equal shares, such as different methods for dividing a square into fourths, to understand that each partition, regardless of shape, represents an equal share of the square. (MP2)
3. Engage in spatial structuring by tiling rectangles with rows and columns of squares to build understanding of two-dimensional regions. (MP7)

Inquiry Questions:

1. How many different triangles can you draw where two of the sides have the same length?
2. (Given a rectangle) Can you divide this rectangle into three equal parts in more than one way?

Coherence Connections:

1. This expectation is in addition to the major work of Grade 2.
2. In Grade 1, students reason with shapes and their attributes, distinguish between defining and non-defining attributes, compose two-dimensional shapes, and partition circles and rectangles into halves and fourths.
3. In Grade 3, students develop understanding of fractions as numbers, use concepts of area and relate area to multiplication and to addition, and understand that shared attributes in different categories of shapes can define a larger category.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

	Unknown Product	Group Size Unknown ("How many in each group? Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red had costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long as the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first.
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here, a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here, a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
,Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here, a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Some examples of situations requiring modeling might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and financial investments.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.